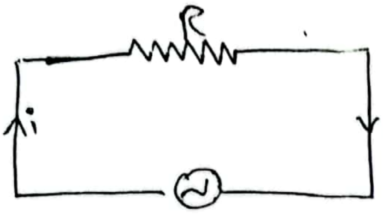
UNIT-3

Steady State Analysis of A.C Circuits

Response to sinusoidal excitation:-

Pure Resistance:-

Consider a simple circuit consisting of pure resistance 'R' ohms connected across a voltage $v = V_m \sin\omega t$. As shown the bellow circuit



$v = V_m \sin\omega t$

According to ohm's law $i = \frac{v}{R}$

$$i = \frac{V_m \sin\omega t}{R}$$

$$i = \frac{V_m}{R}\sin\omega t$$

To compare the above equation with standard equation $i = I_m \sin(\omega t + \phi)$

$$\Rightarrow I_m \sin(\omega t + \phi) = \frac{V_m}{R}\sin\omega t$$

$$\Rightarrow I_m \sin(\omega t) = \frac{V_m}{R}\sin\omega t \qquad (\text{where } \phi = 0)$$

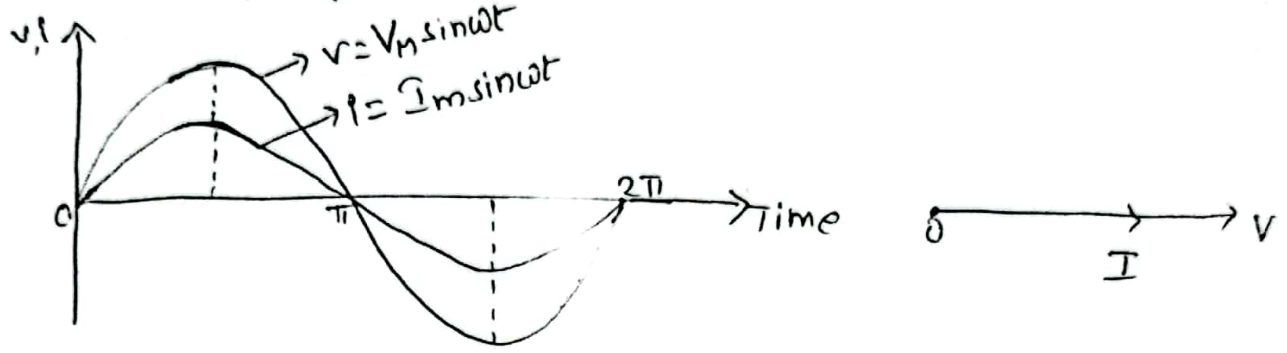
$$\Rightarrow I_m = \frac{V_m}{R}$$

→ Maximum value of alternating current $I_m = \frac{V_m}{R}$

→ when alternating voltage is apply to pure resistance, then the voltage and current are "inphase" each other.

→ The current is going to achieve its maximum (+ve and -ve) and zero whenever voltage is going to achieve its maximum (+ve & -ve) and zero values.

Wave forms of voltage and current, corresponding Phasor diagram



Average power:-

$$P_{Avg} = \frac{1}{T}\int_0^T P(t)\,dt$$

$$P(t) = V(t) * i(t)$$

$$P(t) = V_m \sin\omega t * I_m \sin\omega t$$

$$P(t) = V_m I_m \sin^2\omega t$$

$$P_{Avg} = \frac{1}{\pi}\int_0^{\pi} V_M I_M \sin^2\omega t\, d\omega t$$

$$= \frac{V_M I_M}{\pi}\int_0^{\pi} \sin^2\omega t\, d\omega t$$

$$= \frac{V_M I_M}{2\pi}\int_0^{\pi} [1 - \cos 2\omega t]\, d\omega t$$

$$= \frac{V_M I_M}{2\pi}\left([\omega t]_0^{\pi} - \left[\frac{\sin 2\omega t}{2}\right]_0^{\pi} \right)$$

$$= \frac{V_M I_M}{2\pi}\left[(\pi - 0) - \left(-\frac{\sin 2(\pi)}{2} + \frac{\sin 2(0)}{2} \right) \right]$$

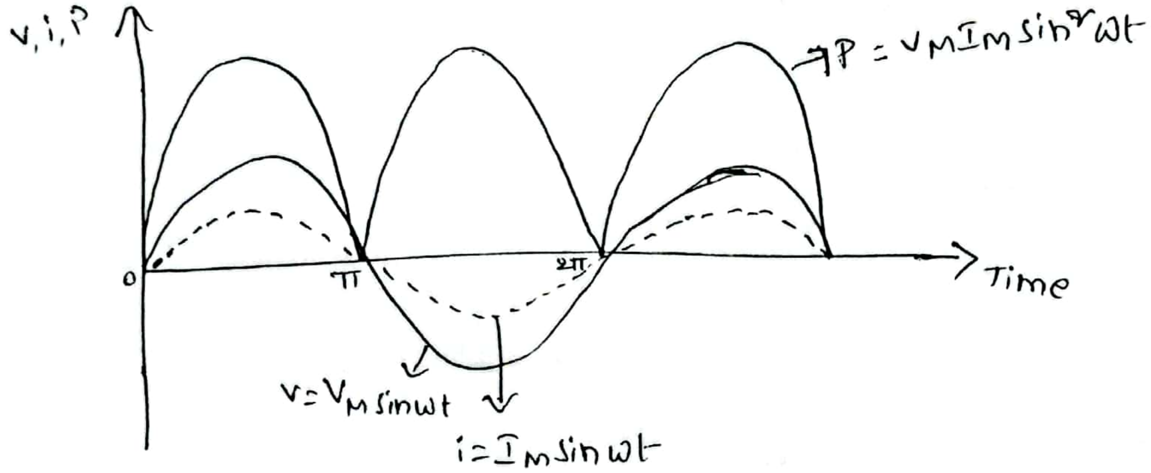
$$P_{Avg} = \frac{V_m I_m}{2\pi}\pi$$

$$= \frac{V_m I_m}{2}$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

$$\boxed{\begin{array}{l} P_{Avg} = V_{rms} \times I_{rms} \text{ watts} \\ P_{Avg} = V \times I \text{ watts} = I^2 R \text{ watts} \end{array}}$$

Voltage, current and power wave forms



→ Average power is equal to product of RMS voltage and RMS current

→ If voltage & currents having time period T. Then the power cycle having time period T/2.

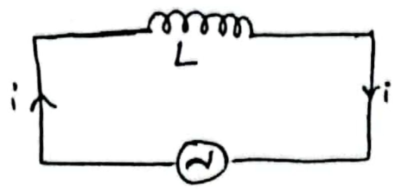
Phasor Diagram:-

$\xrightarrow{I} V$

Impedance:- $Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{R}} = R \quad \therefore \boxed{Z = R}$

Power factor:- $Pf = \cos\phi = \cos(0) = 1$

Pure Inductance:-



$V = V_m \sin\omega t$

consider a simple circuit consists of pure inductance connected across a voltage as shown the above figure.

$$V = L \cdot \frac{di}{dt}$$

$$\Rightarrow V_m \sin\omega t = L \frac{di}{dt}$$

$$\Rightarrow di = \frac{V_m \sin\omega t}{L} dt$$

$$\Rightarrow \int di = \int \frac{V_m}{L} \sin\omega t \, dt$$

$$\Rightarrow \quad i = \frac{V_m}{L}\left(-\frac{\cos\omega t}{\omega}\right)$$

$$= \frac{V_m}{\omega L}(-\cos\omega t)$$

$$= \frac{-V_m}{\omega L} \sin\left(\frac{\pi}{2} - \omega t\right) \qquad \left[\because \cos\omega t = \sin\left(\frac{\pi}{2} - \omega t\right)\right]$$

$$= -\frac{V_m}{\omega L} - \sin(\omega t - \pi/2) \qquad \left[\because \sin(\pi/2 - \omega t) = -\sin(\omega t - \frac{\pi}{2}\right.$$

$$= \frac{V_m}{\omega L} \sin(\omega t - \pi/2)$$

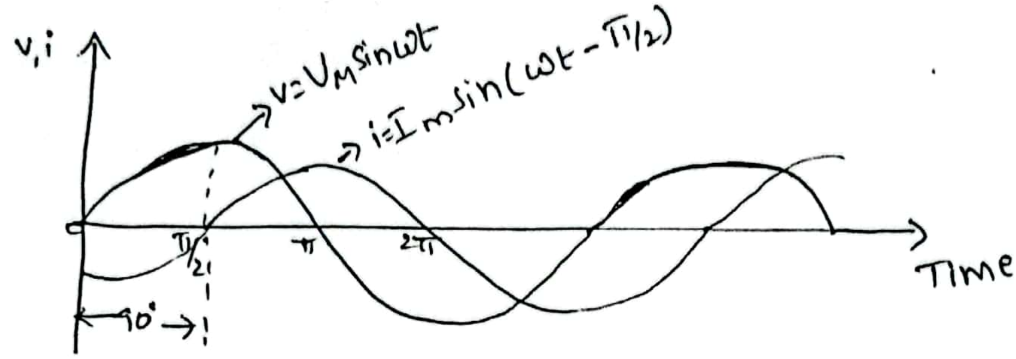
$$i = I_m \sin(\omega t - \pi/2)$$

where $I_M = \frac{V_m}{\omega L} = \frac{V_m}{X_L}$

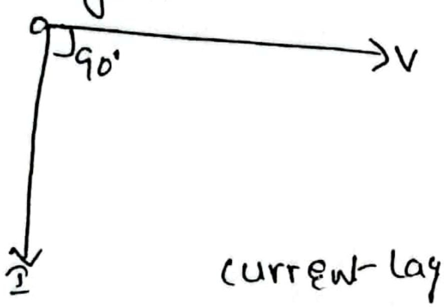
where $X_L = \omega L = 2\pi f L \, \Omega$

→ when alternating voltage is applied to a pure Inductance, then the current lags the voltage by 90°

wave forms and corresponding phasar diagrams



phasor Diagram:-



current lags by 90°

Impedance:-

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\frac{V_m}{\omega L}} = \omega L$$

power factor:-

$$Pf = \cos\phi = \cos(90°) = 0$$

Average power:-

$$P_{Avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$p(t) = v(t) * i(t)$$

$$= V_m \sin\omega t * I_m \sin(\omega t - \pi/2)$$

$$= V_m I_m \sin\omega t \sin(\omega t - \pi/2)$$

$$= V_m I_m \sin\omega t\,(-\sin(\pi/2 - \omega t)) \qquad [\because \sin(\omega t - \pi/2) = -\cos\omega t]$$

$$= V_m I_m \sin\omega t\,(-\cos\omega t)$$

$$= -V_m I_m \sin\omega t \cos\omega t$$

$$= \frac{-V_m I_m}{2}\sin(2\omega t) \qquad (\because 2\sin\omega t\cos\omega t = \sin 2\omega t)$$

$$P_{Avg} = \frac{1}{\pi}\int_0^{\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$

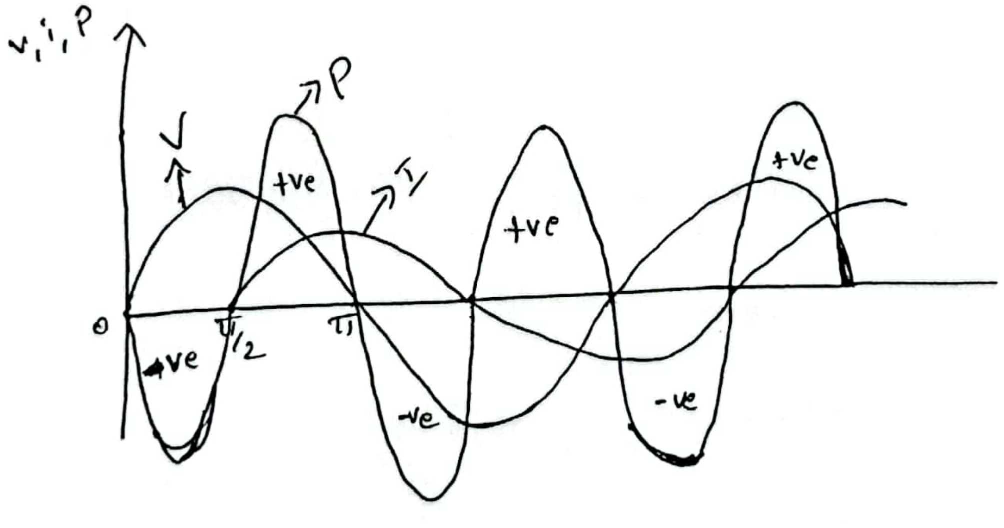
$$= \frac{-V_m I_m}{2\pi}\left[- \frac{\cos 2\omega t}{2}\right]_0^{\pi}$$

$$= \frac{V_m I_m}{2\pi}\left(\frac{\cos 2(\pi)}{2} - \frac{\cos 2(0)}{2}\right)$$

$$= \frac{V_m I_m}{2\pi}\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$\boxed{P_{Avg} = 0}$$

Voltage, current and power wave-forms:-

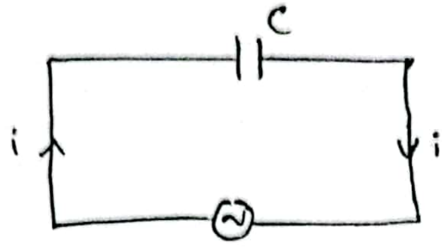


→ when power curve is positive Energy gets stored, due to the increasing current, where as negative power curve this power is return back to the supply.

→ The areas of positive and negative loops are exactly same. So average power consumption is zero.

→ Pure inductance never consumes any power.

Pure capacitance:-



$v = V_m \sin\omega t$

Consider a simple circuit consisting of pure capacitance connected across a voltage as shown the above figure.

$$i = C\frac{dv}{dt}$$

$$i = C\frac{d V_m \sin\omega t}{dt}$$

$$= C V_m \frac{d}{dt}\sin\omega t$$

$$= C V_m \cos\omega t \cdot \omega$$

$$= \omega C V_m \cos\omega t$$

$$= \frac{V_m}{(1/\omega C)} \sin(\omega t + \frac{\pi}{2})$$

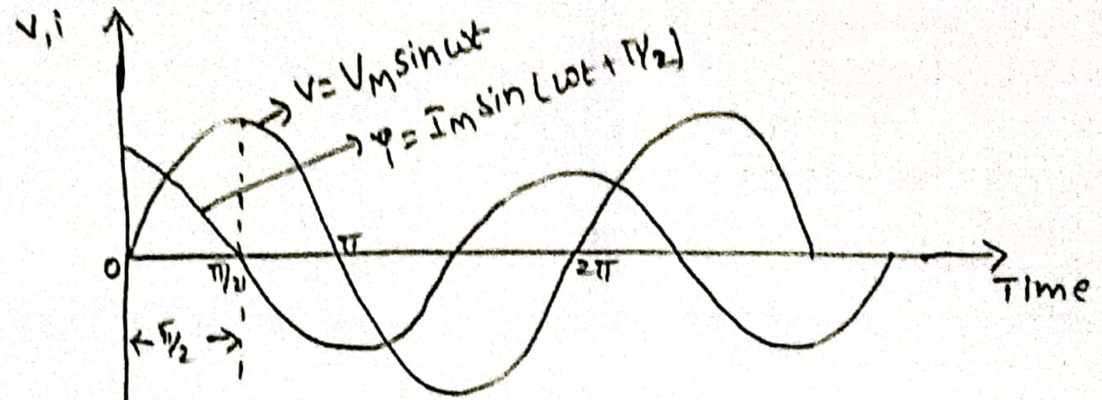
$$i = I_m \sin(\omega t + \pi/2)$$

where $I_m = \frac{V_m}{(1/\omega C)} = \frac{V_m}{X_C}$

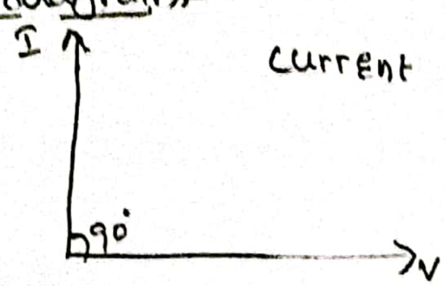
$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}\ \Omega$$

→ when the alternating voltage is applied to a pure capacitance, then the current leads the voltage by 90°

Wave forms and corresponding phasor diagrams



Phasor diagram:-



current Leads by 90°

Impedance:-

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$$

Power factor:-

$$Pf = \cos\phi = \cos(90°) = 0$$

Average power:-

$$P_{Avg} = \frac{1}{T}\int_0^T p(t)\,dt$$

$$p(t) = v(t) * i(t)$$

$$= V_m \sin\omega t * I_m \sin(\omega t + \pi/2)$$

$$= V_m I_m \sin\omega t \sin(\omega t + \pi/2)$$

$$= V_m I_m \sin\omega t \cos\omega t$$

$$p(t) = \frac{V_m I_m}{2}\sin(2\omega t)$$

$$P_{Avg} = \frac{1}{\pi}\int_0^{\pi} \frac{V_m I_m}{2}\sin(2\omega t)\,d\omega t$$

$$= \frac{V_m I_m}{2\pi}\int_0^{\pi}\sin(2\omega t)\,d\omega t$$

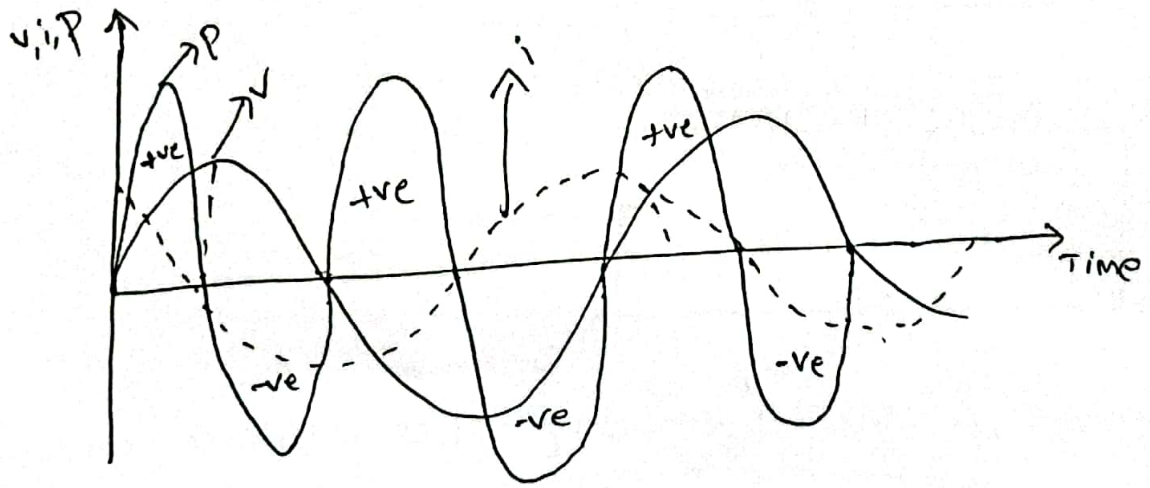
$$P_{Avg} = \frac{V_m I_m}{2\pi}\left[-\frac{\cos 2\omega t}{2}\right]_0^{\pi}$$

$$= -\frac{V_m I_m}{2\pi}\left[\frac{\cos 2(\pi)}{2} - \frac{\cos 2(0)}{2}\right]$$

$$= -\frac{V_m I_m}{2\pi}\left(\frac{1}{2} - \frac{1}{2}\right)$$

$$\boxed{P_{Avg} = 0}$$

Voltage, current and power wave forms :-



→ when power curve is +ve Energy gets stored, due to the increasing current, where as negative power curve this power is return back to the supply

→ The areas of +ve and -ve loops are exactly same. So average power consumption is zero.

→ pure capacitance never consumes any power.

Impedance :- ~~It is~~ defined as ~~the ratio of Vrms and Irms.~~

It is denoted by Z and its units is ohms.

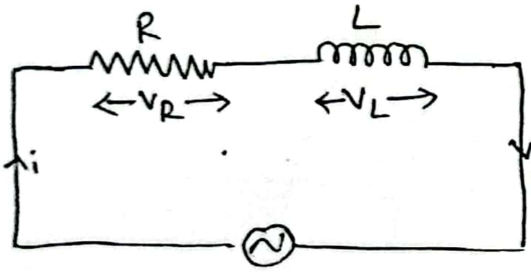
$$Z = \frac{V_{rms}}{I_{rms}} = \frac{V}{I}$$

Def :- It is the ratio of RMS voltage to the RMS current

for Inductor $Z = R + jX_L$

for capacitor $Z = R - jX_C$

A.C through series R-L circuit :-



consider a circuit consisting pure resistance connected in series with a pure inductance. The series combination is connected across a.c supply voltage as shown the above figure.

circuit draws current I and there are two voltage drops

1. Drop across pure resistance $V_R = IR$
2. Drop across pure Inductance $V_L = IX_L$

where $X_L = 2\pi f L$

where I, V_R, V_L are rms values.

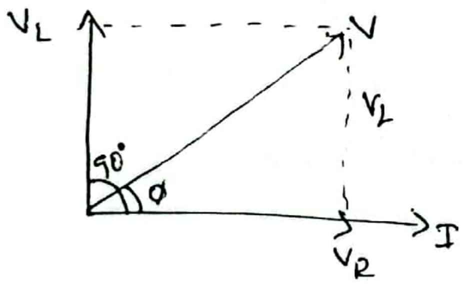
Apply KVL to the circuit

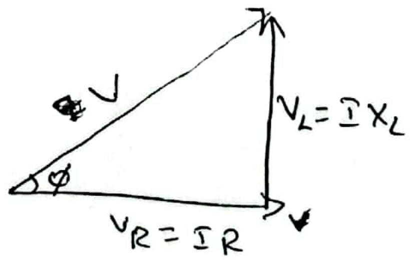
$$\bar{V} = \bar{V}_R + \bar{V}_L \quad (\because \text{phasor addition})$$

$$\bar{V} = \bar{I}R + \bar{I}X_L$$

Steps to draw the phasor diagram:-

1. Take current as a reference phasor.
2. In case of resistance, voltage and current are in phase.
3. In case of Inductance, current lags voltage by 90°. But as current is reference V_L must be leading with respect to current by 90°.
4. supply voltage being vector sum of these two vectors V_L & V_R.





from voltage ∆le

$$V = \sqrt{(V_R)^2 + (V_L)^2}$$

$$= \sqrt{(IR)^2 + (IX_L)^2}$$

$$V = I\sqrt{R^2 + (X_L)^2}$$

$$V = I \cdot Z \qquad \text{where } Z = \sqrt{R^2 + X_L^2}$$

In Rectangular form the impedance is denoted as

$Z = R + jX_L \ \Omega$

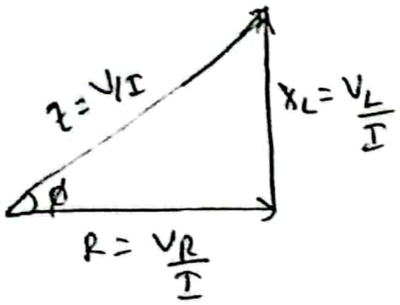
In polar form the impedance is denoted as

$Z = |Z| \angle \phi \ \Omega$

where $Z = \sqrt{R^2 + X_L^2}$ $\phi = \tan^{-1}\left(\frac{X_L}{R}\right)$

Note:- ϕ is +ve for Inductive Impedance

Impedance Δ^{le}



from Impedance Δ^{le} $\sin\phi = \frac{X_L}{Z} = \frac{V_L}{V} \Rightarrow X_L = Z\sin\phi$

$\cos\phi = \frac{V_R}{V} = \frac{R}{Z} \Rightarrow R = Z\cos\phi$

$\tan\phi = \frac{V_L}{V_R} = \frac{X_L}{R}$

Average power:

$P = vi$

$P_{Avg} = V_m \sin\omega t \times I_m \sin(\omega t - \phi)$

$= V_m I_m [\sin(\omega t)\sin(\omega t - \phi)]$

$= \frac{V_m I_m}{2}\left[\cos\phi - \cos(2\omega t - \phi)\right]$

$[\because 2\sin C \sin D = \cos(C-D) - \cos(C+D)]$

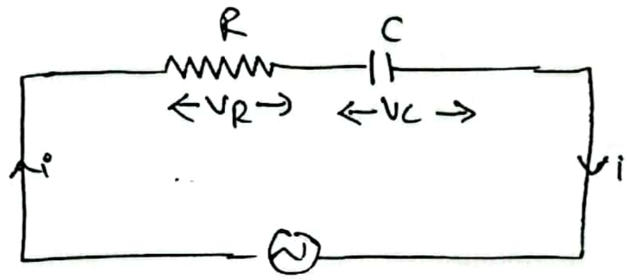
It is also defined as the ratio of resistance to the impedance.

$$\boxed{\frac{R}{Z} = \cos\phi}$$

$$\boxed{\text{Power factor} = \cos\phi}$$

$\cos\phi$ is lagging for Inductive circuit

AC through series R-C circuit:-



$V = V_m \sin\omega t$

consider a circuit consisting of pure resistance connected in series with pure inductance. The series combination is connected across a.c supply voltage as shown the above figure.

circuit draws current I and there are 2 voltage drops

1. Drop across pure resistance $V_R = IR$
2. Drop across pure ~~Inductance~~ capacitance $V_c = I X_C$

where $X_C = \frac{1}{2\pi f C}$

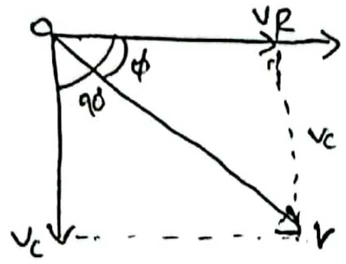
Apply KVL to the circuit

$$\bar{V} = \bar{V_R} + \bar{V_c}$$

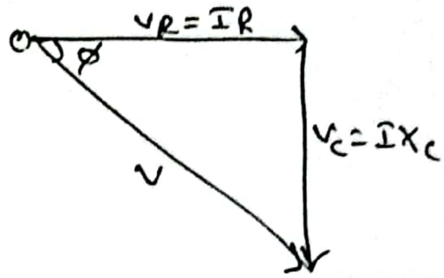
$$\bar{V} = \bar{I}R + \bar{I}X_c$$

Steps to draw the phasor diagram :-

1. Take current as a reference phasor.
2. In case of resistance, voltage and current are in phase.
3. In case of capacitance, current leads voltage by 90°. But as current is reference V_C must be lagging with respect to current by 90°.
4. supply voltage being vector sum of these two vectors V_R & V_C.



voltage Δle



from voltage Δle

$$V = \sqrt{(V_R)^2 + (V_C)^2}$$

$$V = \sqrt{(IR)^2 + (IX_C)^2}$$

$$V = I\sqrt{R^2 + X_C^2}$$

$$V = I.Z \quad \text{where } Z = \sqrt{R^2 + X_C^2}$$

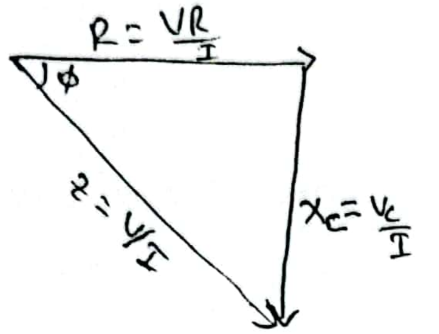
In case of Rectangular form Impedance is denoted as $Z = R + jX_C \ \Omega$

In case of polar form It is denoted as

$Z = |Z| \angle -\phi$ where $|Z| = \sqrt{R^2 + X_C^2}$

$\phi = \tan^{-1}\left(-\frac{X_C}{R}\right)$ φ is -ve for capacitive Impedence

Impedance Δle



from impedance Δle $\sin\phi = \frac{V_c}{V} = \frac{X_c}{Z} \Rightarrow X_c = Z\sin\phi$

$$\cos\phi = \frac{V_R}{V} = \frac{R}{Z} \Rightarrow R = Z\cos\phi$$

$$\operatorname{Tan}\phi = \frac{V_c}{V_R} = \frac{X_c}{R}$$

Average power:-

$$P = v\, i$$

$$P_{Avg} = V_m \sin\omega t * I_m \sin(\omega t + \phi)$$

$$= V_m I_m [\sin\omega t \; I_m \sin(\omega t + \phi)]$$

$$= \frac{V_m I_m}{2}\left[\cos(-\phi) - \cos(2\omega t + \phi\right]$$

$$= \frac{V_m I_m}{2}\cos\phi\phi - \frac{V_m I_m}{2}\cos(2\omega t + \phi) \quad (\because \cos(-\phi) = \cos\phi$$

The second cosine term average value over a cycle is zero.

$$\therefore \; P_{avg} = \frac{V_m I_m}{2}\cos\phi$$

$$P_{Avg} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\phi$$

$$\boxed{P = VI\cos\phi}$$

where V and I are rms values.

If we multiply voltage equation by current-I, we can get the power equation

$$V = V_R + V_C$$

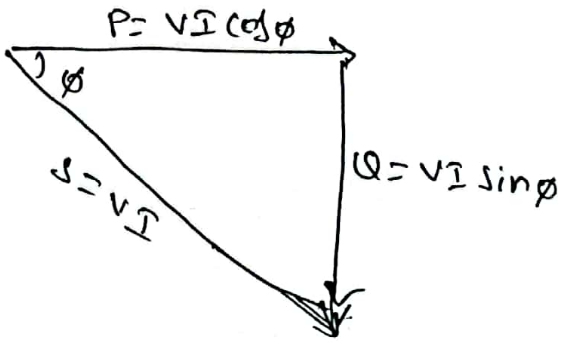
$$VI = V_R I + V_C I$$

$$VI = V\cos\phi I + V\sin\phi I$$

$$VI = VI\cos\phi + VI\sin\phi$$

$$P = VI\cos\phi + VI\sin\phi$$

Power $\triangle^{le}$



apparent power $S = VI$ AV or KVA

True or real power $P = VI\cos\phi$ watts

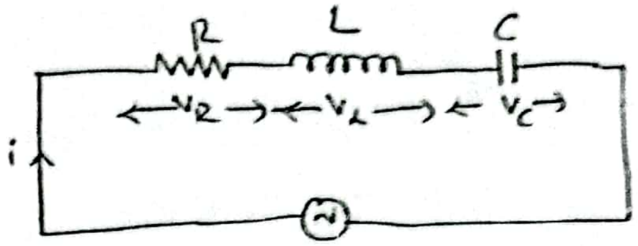
Reactive power $Q = VI\sin\phi$ VAR or KVAR

power factor $= \cos\phi$

- $\cos\phi$ is leading for capacitive circu

AC. Through series RLC Circuit:-

Consider a circuit consisting of resistance, inductance and capacitance connected in series with each other across a.c supply. As shown the bellow circuit.



$v = V_m \sin \omega t$

The circuit draws current I, and there are 3 voltage drops across R, L and C.

Drop across Resistance R is $V_R = IR$

Drop across Inductance $V_L = IX_L$

Drop across capacitance $V_C = IX_C$

According to kirchoff's law

$$\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$$

Steps to draw the phasor diagram:-

1. Take current as reference
2. V_R is inphase with I
3. V_L leads current I by 90°
4. V_C lags current I by 90°
5. obtain the resultant of V_L & V_C. Both V_L & V_C are in phase opposition (180° out of phase)

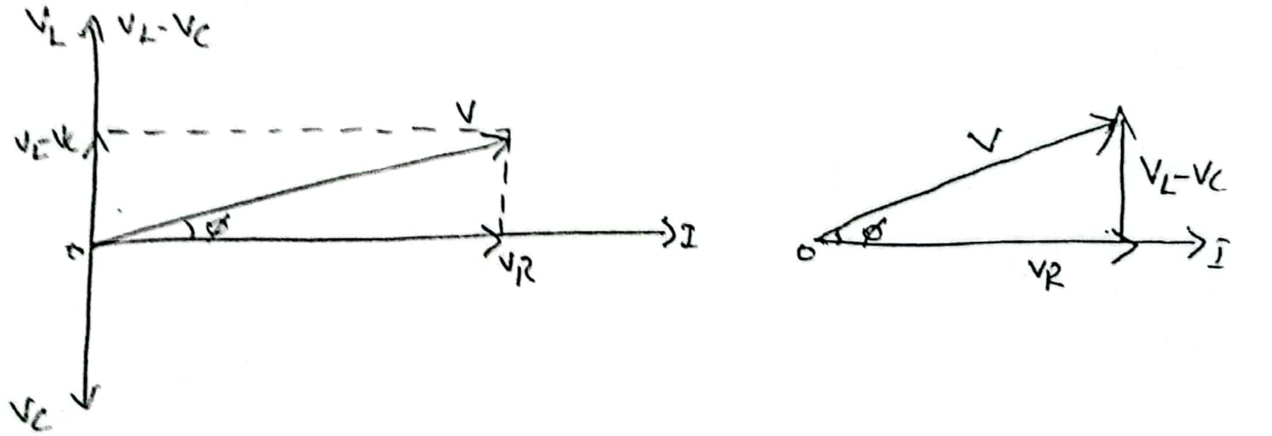
6. Add the resultant of V_L and V_C with V_R by law of Parallelogram to get the supply voltage.

→ The phasor diagram depends on the conditions of the magnitudes of V_L & V_C (or) X_L and X_C (values)

Let us consider the different cases.

1. $\underline{X_L > X_C}$

When $X_L > X_C$ obviously V_L is greater than V_C, so Resultant of V_L and V_C will be directed towards V_L.



Phasor Diagram and voltage traingle for $X_L > X_C$

→ The phasor sum of V_R and $(V_L - V_C)$ gives the resultant supply voltage V.

from the voltage △le

$$V = \sqrt{(V_R)^2 + (V_L - V_C)^2}$$

$$= \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$= IZ$$

where $Z = \sqrt{R^2 + (X_L - X_C)^2}$

(18)

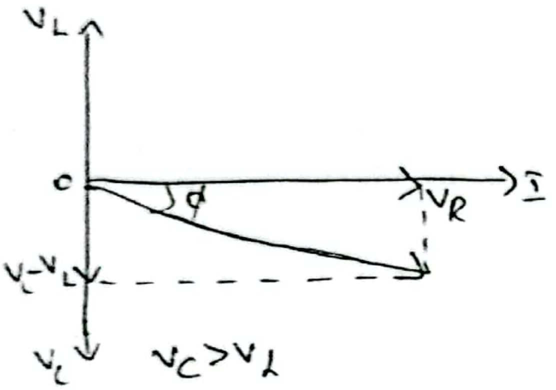
In this condition if $v = V_m \sin \omega t$ then

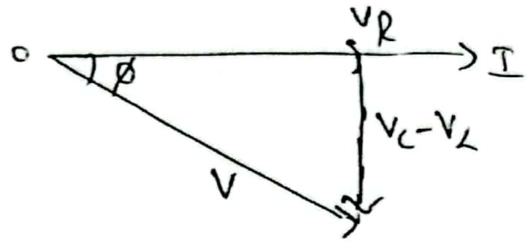
$i = I_m \sin(\omega t - \phi)$ as current lags voltage by angle ϕ.

2. $X_L < X_C$

when $X_L < X_C$, obviously V_L is less than V_C, so the resultant of V_L and V_C will be directed towards V_C.

phasor diagram and voltage traingle for $X_L < X_C$





The phasor sum of V_R and $(V_C - V_L)$ gives the resultant supply voltage v.

from voltage Δ'le

$$V = \sqrt{(V_R)^2 + (V_C - V_L)^2}$$

$$= \sqrt{(IR)^2 + (IX_C - IX_L)^2}$$

$$= I\sqrt{R^2 + (X_C - X_L)^2}$$

$$= IZ$$

where $Z = \sqrt{R^2 + (X_C - X_L)^2}$

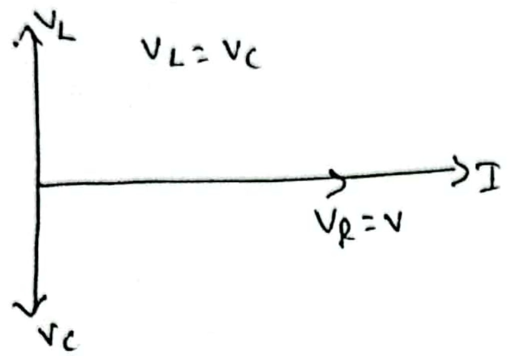
In this condition, if $v = V_m \sin \omega t$ then

$i = I_m \sin(\omega t + \phi)$ as current leads voltage by angle ϕ

3 $X_L = X_C$:-

when $X_L = X_C$, obviously $V_L = V_C$ so V_L and V_C will cancel each other and their resultant is zero. in this case $V_R = V$ and the circuit acts as purely resistive circuit.

phasor diagram for $X_L = X_C$



from phasor Diagram

$V = V_R$

$V = IR$

$V = IZ$

$RI = IZ$

where $Z = R$

Impedance:-

for RLC series circuit impedance is give by

$Z = R + jX$

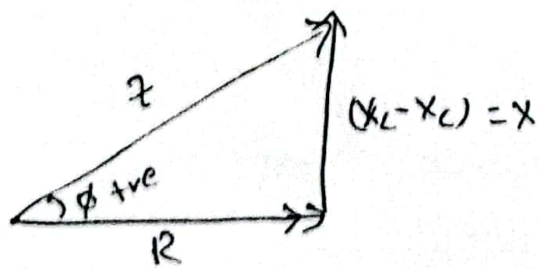
where $X = X_L - X_C$ = Total reactance of circuit.

If $X_L > X_C \rightarrow X$ is positive and circuit is inductive

if $X_L < X_C \rightarrow X$ is Negative and circuit is capacitive.

if $X_L = X_C \rightarrow X$ is zero and circuit is purely resistive

for $X_L > X_C$ ϕ is positive, the impedance traingle is shown bello

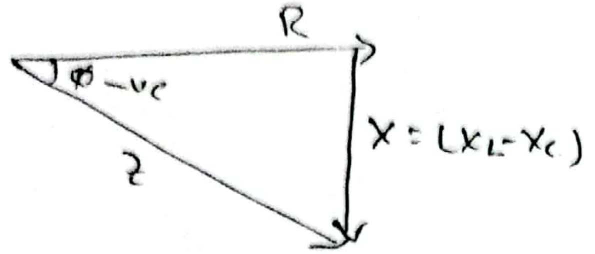


$\sin\phi = \frac{X}{Z}$

$\cos\phi = \frac{R}{Z}$

$\tan\phi = \frac{X}{R}$

For $X_L < X_C$, ϕ is negative. the impedance traingle is shown bellow



$\sin\phi = \frac{X}{Z}$

$\cos\phi = \frac{R}{Z}$

$\tan\phi = \frac{X}{R}$

Power and Powe Traingle:-

P_{Avg} = Average power consumed by 'R' + Average paer consumed by 'L' + average paer consumed by 'C'

But pure Inductance and capacitance never consumes any powen

$\therefore P_{Avg}$ = Power Taken by R

$= I^2 R$

$= I(IR)$

$= I V_R$

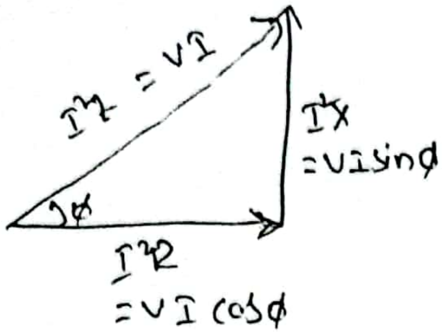
$\boxed{P = VI\cos\phi}$ W $V_R = V\cos\phi$ In both cases

for any condition $X_L > X_C$ or $X_L < X_C$ the power can be expressed as

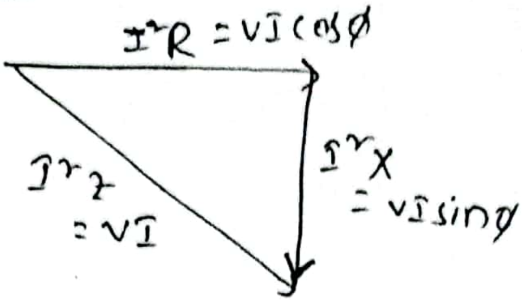
$\boxed{P = \text{voltage} \times \text{component of current in phase with voltage}}$

→ The power Traingle can be obtained by multiplying each side of impedance traingle by I^2.

for $X_L > X_C$



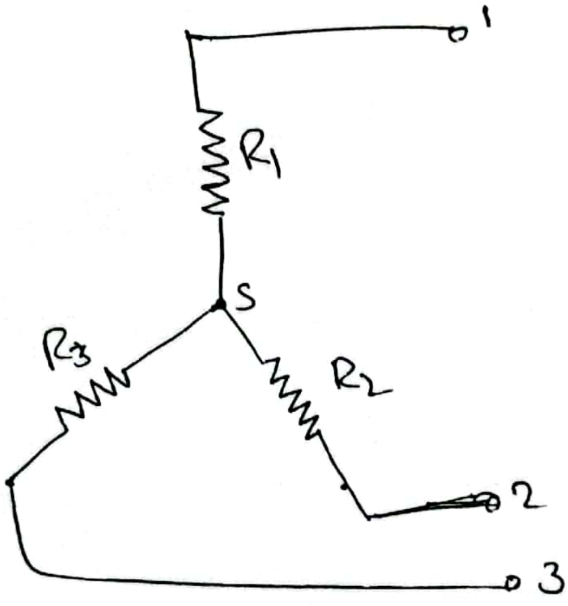
for $X_L < X_C$



Star connection:-

In star connection three resistances are connected as that one end of each resistance is connected together. It form a junction point is called star point.

Star connection of three resistances

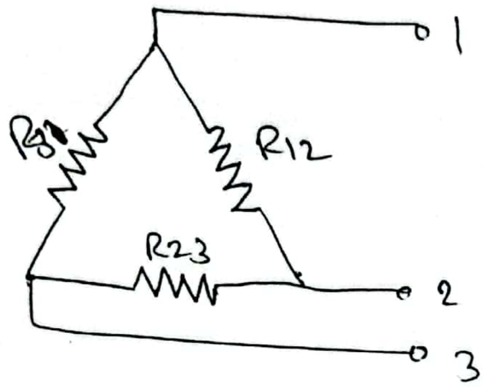


Delta connection:-

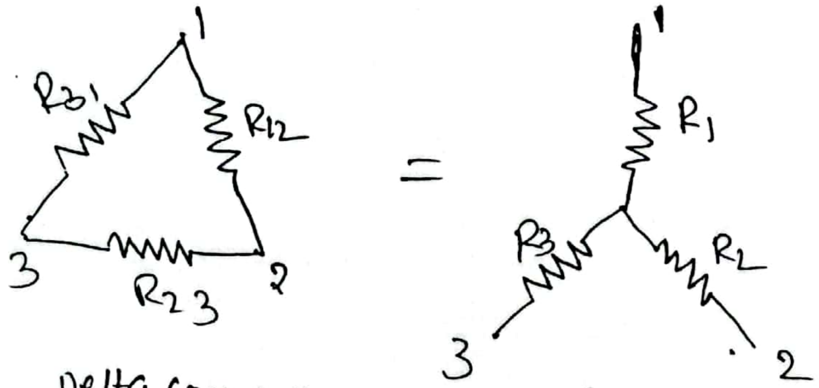
In delta connection three resistances are connected as that one end of the first resistor is connected to first end of the second resistor, the second end of second resistor is connected to first end of third resistor and so on to complete a loop.

Delta connection always forms a loop or closed path.

Delta connection of three resistances



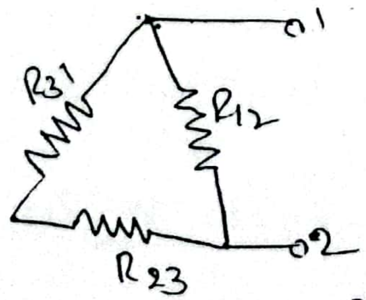
Delta - star Transformation:-



Delta connection   Equivalent star connection.

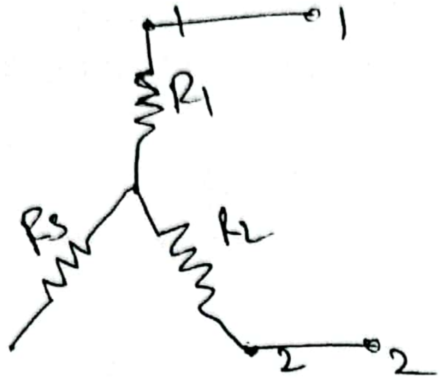
consider three resistances R_{12}, R_{23} & R_{31} conneted in delta connection as shown the above figure.

Consider the terminals ① & ②



$$R_{eq} = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} \longrightarrow ①$$

consider same two terminals in equivalent star connection



$$R_{eq} = R_1 + R_2 \longrightarrow (2)$$

Equate the equation (1) & (2)

$$R_1 + R_2 = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+(R_{23}+R_{31})} \longrightarrow (3)$$

Similarly

$$R_2 + R_3 = \frac{R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}} \longrightarrow (4)$$

$$R_3 + R_1 = \frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} \longrightarrow (5)$$

Subtracting equation (3) from equation (4)

$$R_1+R_2-(R_2+R_3) = \frac{R_{12}(R_{23}+R_{31})}{R_{12}+R_{23}+R_{31}} - \frac{R_{23}(R_{31}+R_{12})}{R_{12}+R_{23}+R_{31}}$$

$$R_1+\cancel{R_2}-\cancel{R_2}-R_3 = \frac{R_{12}\cancel{R_{23}}+R_{12}R_{31}-R_{23}R_{31}-\cancel{R_{23}}R_{12}}{R_{12}+R_{23}+R_{31}}$$

$$R_1 - R_3 = \frac{R_{12}R_{31} - R_{23}R_{31}}{R_{12}+R_{23}+R_{31}} \longrightarrow (6)$$

Adding equation (6) & (5)

$$R_1 + R_3 + R_1 - R_3 = \frac{R_{31}(R_{12}+R_{23})}{R_{12}+R_{23}+R_{31}} + \frac{R_{12}R_{31} - R_{23}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$2R_1 = \frac{R_{31}R_{12} + \cancel{R_{31}R_{23}} + R_{12}R_{31} - \cancel{R_{23}R_{31}}}{R_{12}+R_{23}+R_{31}}$$

$$2R_1 = \frac{R_{31}R_{12} + R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

$$\cancel{2}R_1 = \frac{\cancel{2}R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

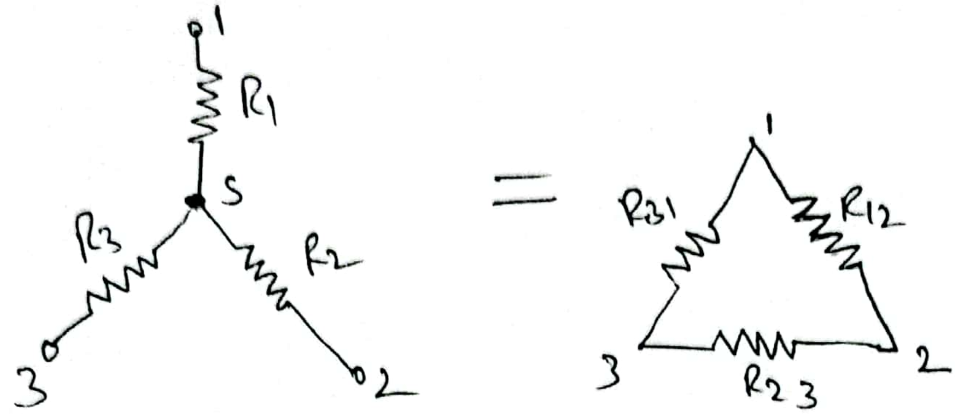
$$R_1 = \frac{R_{12}R_{31}}{R_{12}+R_{23}+R_{31}}$$

similarly

$$R_2 = \frac{R_{23}R_{12}}{R_{12}+R_{23}+R_{31}}$$

$$R_3 = \frac{R_{31}R_{23}}{R_{12}+R_{23}+R_{31}}$$

Star to Delta Transformation:-



Consider 3 resistances R_1, R_2 and R_3 connected in sta connection between the terminals 1, 2, and 3.

from Delta to star transformation we know that

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \longrightarrow (1)$$

$$R_2 = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} \longrightarrow (2)$$

$$R_3 = \frac{R_{15} R_{23}}{R_{12} + R_{23} + R_{31}} \longrightarrow (3)$$

To multiply the equations (1) & (2), (2) & (3) and (3) & (1)

$$R_1 R_2 = \frac{R_{12}^2 R_{23} R_{31}}{(R_{12} + R_{23} + R_{31})^2} \longrightarrow (4)$$

$$R_2 R_3 = \frac{R_{23}^2 + R_{12} + R_{13}}{(R_{12} + R_{23} + R_{31})^2} \longrightarrow (5)$$

$$R_3 R_1 = \frac{R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2} \longrightarrow (6)$$

To add the equation (4), (5) & (6)

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2 R_{23} R_{31}}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{23}^2 R_{31} R_{12}}{(R_{12}+R_{23}+R_{31})^2} + \frac{R_{31}^2 + R_{12} + R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12}^2 R_{23} R_{31} + R_{23}^2 R_{31} R_{12} + R_{31}^2 + R_{12} + R_{23}}{(R_{12}+R_{23}+R_{31})^2}$$

$$\cancel{R_1R_2 + R_2R_3 + R_3R_1} = \cancel{(R_{12} + R_{23} + R_{31})(R_{12} - R_{23} - R_{31})}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{23} R_{31} (R_{12} + R_{23} + R_{31})}{(R_{12}+R_{23}+R_{31})^2}$$

$$R_1R_2 + R_2R_3 + R_3R_1 = \frac{R_{12} R_{23} + R_{31}}{R_{12}+R_{23}+R_{31}}$$

From equation (1) $R_1 = \dfrac{R_{12} R_{31}}{R_{12}+R_{23}+R_{31}}$

Substituting R_1 value in above equation

$$R_1R_2 + R_2R_3 + R_3R_1 = R_1 R_{23}$$

$$R_{23} = \frac{R_1R_2}{R_1} + \frac{R_2R_3}{R_1} + \frac{R_3R_1}{R_1}$$

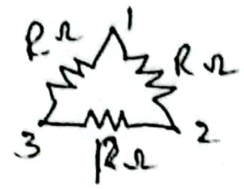
$$\boxed{R_{23} = R_2 + R_3 + \frac{R_2R_3}{R_1}}$$

similarly

$$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$

$$R_{31} = R_3 + R_1 + \frac{R_1 R_3}{R_2}$$

→ If all resistances in a delta connection have same magnitude (resistances) say R, then its equivalent star will contain



$$\boxed{R_1 = R_2 = R_3 = \frac{R \times R}{R+R+R} = R/3}$$

→ If all three resistances in a star connection have same magnitude (resistances) say R, then its equivalent Delta connection will contains

$$\boxed{R_{12} = R_{23} = R_{31} = R + R + \frac{R \times R}{R} = 3R}$$

Loop analysis or Mesh Analysis:-

This method is useful for the circuits that have many nodes and loops. It is mainly based on KVL.

Steps for the loop or mesh Analysis:-

step1: choose the various loops

step2: Show the various loop currents and the polarities of associated voltage drops.

UNIT - III

coupled circuits:-

Self inductance:- If the current i and flux linkages φ refer to the same physical system, then the parameter L is called self inductance.

* when a current changes in a circuit, the magnetic flux linking the same circuit changes and an e.m.f is induced in the circuit.

This induced emf is proportional to the rate of change current.

$$v = L\frac{di}{dt} \rightarrow ①$$

where, v = induced voltage

$\frac{di}{dt}$ = rate of change of curret

L = constant of proportionality called self inductance units: H

This inductance is also expressed as

$$L = \frac{N\phi}{i}$$

Where N = No of turns in the ckt

ϕ = flux linkage

$$\Rightarrow i = \frac{N\phi}{L} \rightarrow ②$$

Substitution of eqn ② in ①

$$v = L\frac{d\left(\frac{N\phi}{L}\right)}{dt}$$

$$v = L \times \frac{1}{L} \times N\frac{d\phi}{dt}$$

$$v = N\frac{d\phi}{dt} \rightarrow ③$$

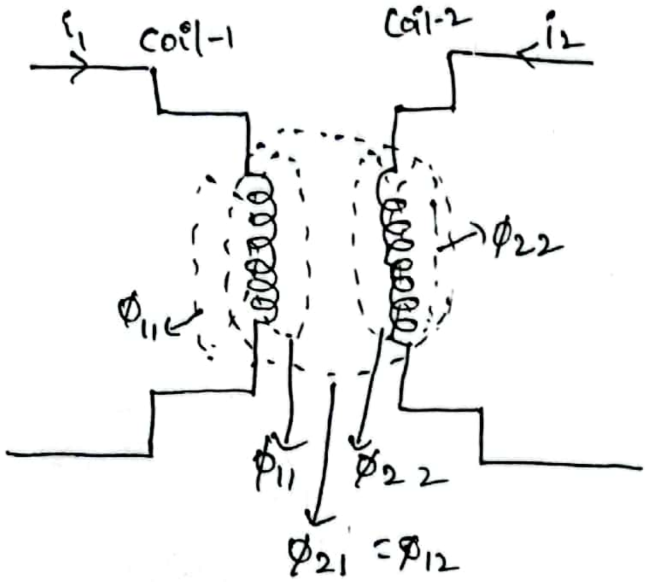
Comparing eqn ① and eqn ③

$$L\frac{di}{dt} = N\frac{d\phi}{dt}$$

$$\cancel{L = N\frac{d\phi}{dt}} \quad L = N\frac{d\phi}{di} \rightarrow \text{eq}^n ④$$

equation ④ represents the magnetude of self inductance

Mutual inductance:-



N_1 = No of turns in coil-1
N_2 = No of turns in coil-2

* Let two coils carry currents i_1 and i_2
* Each coil will have linkage flux ϕ_{11} and ϕ_{22} for coil-1 and coil-2 respectively.
* As well as mutual flux ϕ_{21}. the flux of coil-2 links with coil-1 or ϕ_{12}. the flux of coil-1 links with coil-2

The induced voltage of coil-2 is given by

$$V_{L_2} = N_2\frac{d\phi_{12}}{dt} \rightarrow ①$$

Since ϕ_{12} is related to the current of coil-1 and the induced voltage is proportional to the rate of change of i_1

$$V_{L_2} = M \cdot \frac{di_1}{dt} \rightarrow ②$$

where M is constant ~~tetmect~~ of proportionality termed as mutual inductance between the two coils.

Compare eqn ①&②
$$M\frac{di_1}{dt} = N_2\frac{d\phi_{12}}{dt}$$

$$M = N_2\frac{d\phi_{12}}{di_1}$$

Similarly
$$M = N_1\frac{d\phi_{21}}{di_2}$$

Coefficient of coupling:-

It is defined as the fraction of total flux that links the coils.

i.e k = coefficient of coupling $= \frac{\phi_{12}}{\phi_1} = \frac{\phi_{21}}{\phi_2}$

since, $\phi_{12} < \phi_1$ and $\phi_{21} < \phi_2$

hence the maximum value of k is unity.

The Mutual inductance equation are

$$M = N_2\frac{\phi_{12}}{i_1} \rightarrow ①$$

$$M = N_1\frac{\phi_{21}}{I_2} \rightarrow ②$$

Multiplying Equations ① & ②, we get

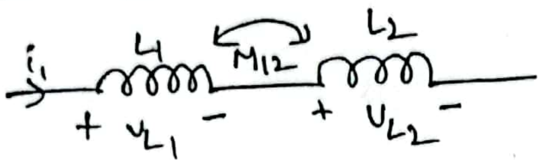
$$M^2 = N_1 N_2 \frac{\phi_{21}\phi_{12}}{i_1 i_2} = N_1 N_2 \frac{k\phi_1 \cdot k\phi_2}{i_1 \cdot i_2} \quad \left[\because \phi_{21} = k\phi_1, \ \phi_{12} = k\phi_2\right]$$

$$= k^2 N_1 \frac{\phi_1}{i_1} \cdot N_2 \frac{\phi_2}{i_2}$$

$$= k^2 L_1 L_2 \quad \left[\because L = \frac{N\phi}{i}\right]$$

$$M = k\sqrt{L_1 L_2}$$

Series connection of coupled coils:-



Let us consider two coils of self inductance L_1 and L_2 are connected in series such that the voltage induced in coil-1 is v_{L_1} and that in coil-2 is v_{L_2}

While a current i flows through them.

Let M_{12} be the mutual inductance.

$$v_{L_1} = L_1 \frac{di}{dt} + M_{12}\frac{di}{dt} = (L_1 + M_{12})\frac{di}{dt}$$

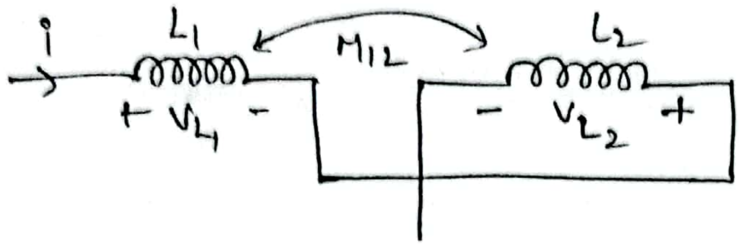
$$v_{L_2} = L_2 \frac{di}{dt} + M_{12}\frac{di}{dt} = (L_2 + M_{12})\frac{di}{dt}$$

$\therefore$ Net voltage $v_L = v_{L_1} + v_{L_2} = (L_1 + M_{12})\frac{di}{dt} + (L_2 + M_{12})\frac{di}{dt}$

$$v_L = \frac{di}{dt}\left[L_1 + M_{12} + L_2 + M_{12}\right]$$

$$v_L = (L_1 + L_2 + 2M)\frac{di}{dt} \quad (\because M_{12} = M)$$

If the coils are still series connected but the flux of both the coils oppose each other. as shown bellow figure.



$$\therefore V_{L_1} = (L_1 - M_{12})\frac{di}{dt}$$

$$V_{L_2} = (L_2 - M_{12})\frac{di}{dt}$$

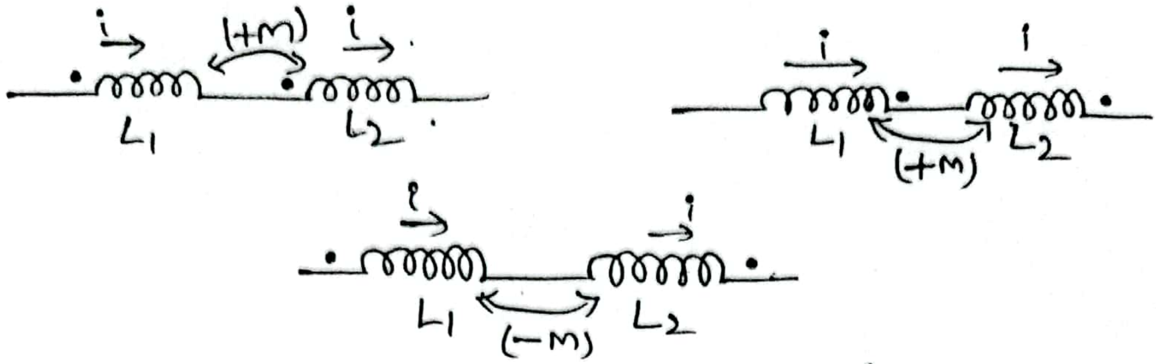
$$\therefore V_L = (L_1 + L_2 - 2M_{12})\frac{di}{dt}$$

$$V_L = (L_1 + L_2 - 2M)\frac{di}{dt}$$

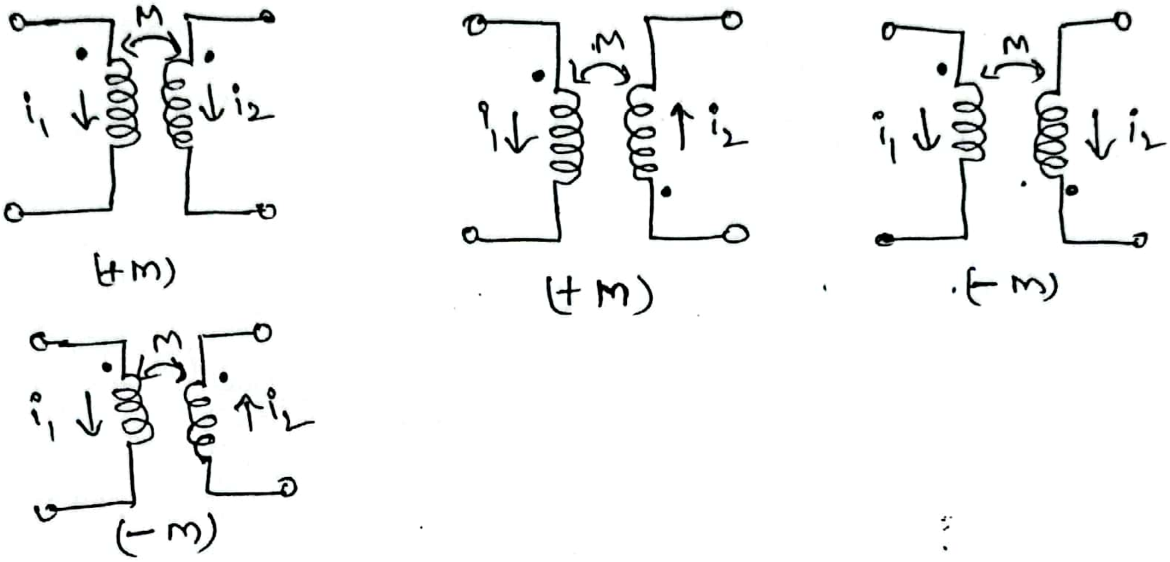
$\therefore$ The net inductance $L = (L_1 + L_2 - 2M)$

Dot convention in coupled coils:-

* To determine the relative polarity of the induced voltage in the coupled coil, the coils are marked with Dots.
* when the currents through each of the mutually coupled coils are going away from the dot or towards the dots then the mutual inductance is +ve.
* when the current entering the one coil and leaving the dot from other coil, then the mutual inductance is -ve. as shown bellow figures.

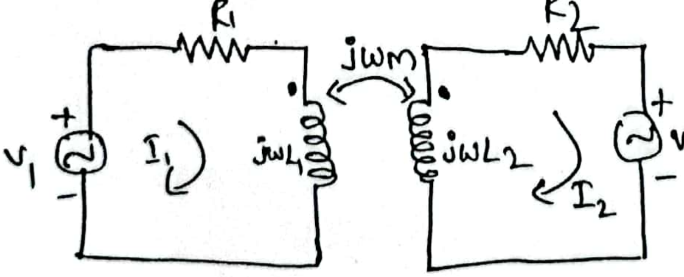


The dot convention of few possibilities of mutually coupled transformer coils are indicated below

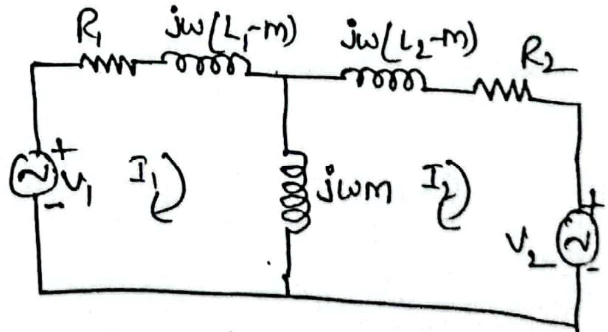


Conductively coupled Equivalent circuits:-

It is possible in analysis to replace a mutually coupled circuit with a conductively coupled circuit. as shown bellow figures.



Mutual coupled circuit



equivalent circuit

Let I_1 and I_2 be the loop current, the loop equations of mutual coupled circuit are

$$V_1 = R_1 I_1 + j\omega L_1 I_1 - \cancel{j\omega M(I_1 - I_2)}\ j\omega M I_2$$

$$V_1 = (R_1 + j\omega L_1) I_1 - j\omega M I_2$$

$$V_2 = R_2 I_2 + j\omega L_2 I_2 - j\omega M I_1$$

$$V_2 = (R_2 + j\omega L_2) I_2 - j\omega M I_1$$

The loop equations for the equivalent circuit are

for Loop1:- $I_1(R_1 + j\omega(L_1 - M)) + (I_1 - I_2) j\omega M = V_1$

$$I_1(R_1 + j\omega L_1) - I_1 j\omega M + I_1 j\omega M - I_2 j\omega M = V_1$$

$$\therefore V_1 = I_1(R_1 + j\omega L_1) - j\omega M I_2$$

For Loop2:- $(I_2 - I_1) j\omega M + I_2(R_2 + j\omega(L_2 - M)) = -V_2$

$$V_2 = +j\omega M I_2 - j\omega M I_1 + I_2(R_2 + j\omega L_2) - I_2 j\omega M$$

$$V_2 = -j\omega M I_1 + I_2(R_2 + j\omega L_2)$$

The voltage equations of mutual coupled circuit and equivalent circuits are identical.